On the Weibull Autocorrelation Function: Field Trials and Validation

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***Abstract*— Indoor and outdoor field trial measurements are used to validate the autocorrelation function derived in an exact manner for the Weibull fading signal. Comparisons are performed and an *excellent* fitting to the field measurements have been found. Moreover, in order to explore the first-order statistics, the cumulative density function (CDF) was also computed for the same cases.**

***Index Terms*— Field trials, Weibull autocorrelation function, Weibull distribution, validation.**

I. Introduction

The performance of the wireless channel is strongly affected by the multipath fading phenomena. In order to mitigate this effect, a deep knowledge of the characteristics and correct modeling of fading channels is imperative. Many statistical models have been used to describe the multipath fading phenomenon [1]. Some of these models produce very accurate results, especially the Rice and Nakagami-m distributions [2]. Another useful model is Weibull, which was first used in problems dealing with reliability. Indeed, the Weibull distribution is a simple and flexible statistical model for describing multipath fading phenomena, for both indoor and outdoor propagation environments.

Experimental data supporting the Weibull fading model have been reported in [3]. Indoor and outdoor applications of the Weibull model were considered in [4] and [5], respectively. In [6], the Weibull and Nakagami-*m* models were recommended for theoretical studies as the introduce slope changes in the distribution tail, which compensates for shortcomings of the Rayleigh model. In [7], measurements revealed that the Weibull distribution had the best fit to path-loss models of the narrow-band digital enhanced cordless telecommunications (DECT) system at reference frequency 1.89 GHz.

A substantial portion of the literature dealing with field measurements in Weibull fading channels has been devoted to the study of the first order statistics. Few works investigate the high order statistics of the Weibull channel model. In [8], the level crossing rate (LCR) and the average fade duration (AFD) of the Weibull channel have been obtained, whereas in [9] these statistics have been attained for the diversity-combined case. Very recently [10], a simple-closed form expression for the generalized cross-moments of the Weibull distribution has been derived. From this expression, the derivation of the autocorrelation function follows directly.

In this paper, the autocorrelation function derived in [10] is validated through field measurements. Comparisons have been performed and an *excellent* fitting to the field measurements have been found. Moreover, in order to explore the first-order statistics, the cumulative density function (CDF) was also computed for the same cases.

II. The Autocorrelation Function

The temporal autocorrelation function $A_R(\tau)$ of the Weibull envelope R has been recently obtained in [10] as

$$A_R(\tau) \triangleq E[R(t)R(t+\tau)] = \hat{r}^2\,\Gamma^2\left(1+\frac{1}{\alpha}\right)\,{}_2F_1\left(-\frac{1}{\alpha},-\frac{1}{\alpha};1;J_0^2(\omega_D\tau)\right) \quad (1)$$

where $\hat{r} = \sqrt[\alpha]{E[R^\alpha]}$ is the α-root mean value of R^α, $E[\cdot]$ denotes the expectation operator, α is the Weibull parameter, $\Gamma(.)$ is the Gamma function [11, Eq. 8.310.1], ${}_2F_1(.)$ is the Gauss hypergeometric function [11, Eq. 9.14.1], $J_0(.)$ is the Bessel function of the first kind and zeroth order [11, Eq. 8.401], and ω_D is the maximum Doppler shift given in rad/s.

Using the space-time duality of the wireless channel [12], it is readily known that $\omega_D\tau = 2\pi d/\lambda$, where d is the distance between antennas, and λ is the carrier wavelength. Then, the spacial autocorrelation function $A_R(d)$ of R is

$$A_R(d) = \hat{r}^2\,\Gamma^2\left(1+\frac{1}{\alpha}\right)\,{}_2F_1\left(-\frac{1}{\alpha},-\frac{1}{\alpha};1;J_0^2(2\pi d/\lambda)\right) \quad (2)$$

A. The moment-based α-estimator

The moments of the Weibull envelope are given as [10]

$$E[R^k] = \hat{r}^k\Gamma(1+k/\alpha) \quad (3)$$

From (3), it follows that

$$\frac{E^i[R^j]}{E^j[R^i]} = \frac{\Gamma^i\left(1+j/\alpha\right)}{\Gamma^j\left(1+i/\alpha\right)} \tag{4}$$

For a particular case in which $i = 2$ and $j = 1$, (4) yields

$$\frac{E^2[R]}{E[R^2]} = \frac{\Gamma^2\left(1+1/\alpha\right)}{\Gamma\left(1+2/\alpha\right)} \tag{5}$$

Note that the estimator presented in (5) is given in terms of the ratio of the squared first and second moments. Of course, from (4) there are other moment-based estimators, however, the one presented in (5) is given by the lowest integer order.

Given a set of measured data for the fading envelope R, the practical procedure in order to determine the distribution parameter α is to find the root of the transcendental equation (5). In fact, this method provides a simple and low-complexity parameter estimator.

III. Field Trials and Validation

A series of field trials was conducted at the University of Campinas (Unicamp), Brazil, in order to validate the autocorrelation function of the Weibull envelope. The transmitter was placed on the rooftop of one of the buildings and the receiver travelled through the campus as well as within the buildings. The mobile reception equipment was especially assembled for this purpose. Basically, the setup consisted of a vertically polarized omnidirectional receiving antenna, a low noise amplifier, a spectrum analyzer, data acquisition apparatus, a notebook computer, and a distance transducer for carrying out the signal sampling. The transmission consisted of a CW tone at 1.8 GHz. The spectrum analyzer was set to zero span and centered at the desired frequency, and its video output used as the input of the data acquisition equipment with a sampling rate of $\lambda/14$. The local mean was estimated by the moving average method, with the average being conveniently taken over samples symmetrically adjacent to every point. From the data collected, the long term fading was filtered out and the Weibull parameter α, as defined previously, was estimated.

The normalized empirical autorrelation was computed according to

$$\widehat{A}_R\left(\Delta\right) = \frac{\sum_{i=1}^{N-\Delta} r_i r_{i+\Delta}}{\sum_{i=1}^{N} r_i^2} \tag{6}$$

where r_i is the i-th sample of the amplitude sequence, N is the total number of samples, Δ is the discrete relative distance difference, and $\widehat{A}_R(.)$ denotes an empirical average of $A_R(.)$.

The empirical autocorrelation function was compared against the corresponding theoretical formula (2) and plotted as a function of d/λ with the same parameter α estimated from the experimental data. Furthermore, a numerical measure of the mean error deviation[1], ϵ, was computed for each case. Figs. 1, 2, and 3 show some sample plots comparing the experimental and theoretical autocorrelation data for different values of α. Note the *excellent* fitting and how the theoretical curve tends to keep track of the change of concavity of the empirical data. As can be observed, in the three cases the error deviation were smaller than 2%.

Moreover, in order to explore the first and the second order statistics, the CDFs[2] were also computed for the same cases of the autocorrelation functions. Figs. 4, 5, and 6 show that the theoretical Weibull CDFs fit our experimental results very well. For comparison purposes, the Rayleigh CDFs are also shown. In all of the cases, the Weibull distribution provides the best fitting, mainly in the tail.

IV. Conclusions

In this paper we have reported the results of field trials aimed at investigating the first and second-order statistics of short term fading signals. It has been found an *excellent* agreement between the experimental and the theoretical data. The measurements validate the autocorrelation formula derived in an exact manner in [10] for the Weibull fading signal.

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[1] The mean error deviation between the measured data x_i and the theoretical value y_i is defined as $\epsilon = \frac{1}{N}\sum_{i=1}^{N}\frac{|y_i - x_i|}{x_i}$, where N is the number of points.

[2] The normalized Weibull CDF is given by $F_{\mathrm{P}}\left(\rho\right) = 1 - \exp\left(-\rho^{\alpha}\right)$, where $\mathrm{P} = R/\hat{r}$ is the normalized envelope.

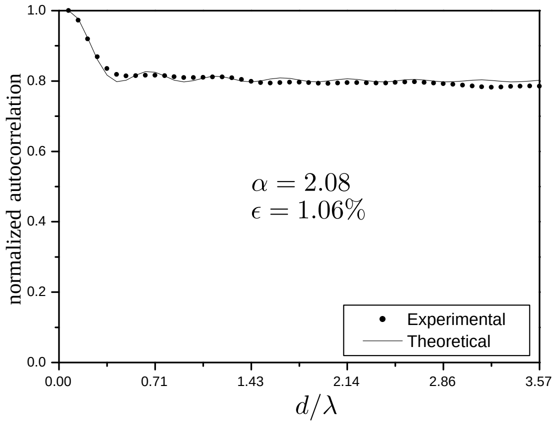

Fig. 1. Empirical versus theoretical autocorrelation function

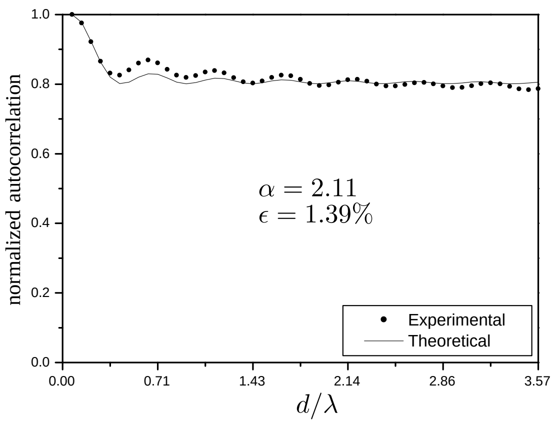

Fig. 2. Empirical versus theoretical autocorrelation function

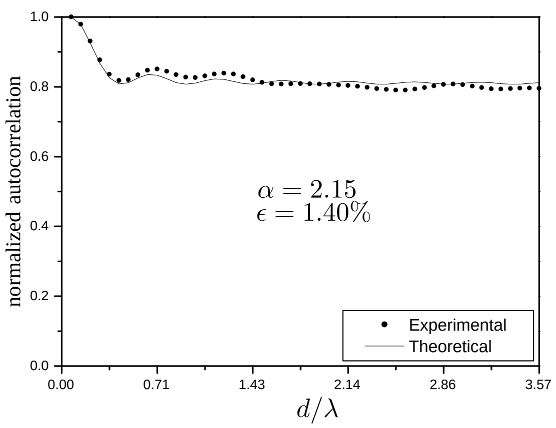

Fig. 3. Empirical versus theoretical autocorrelation function

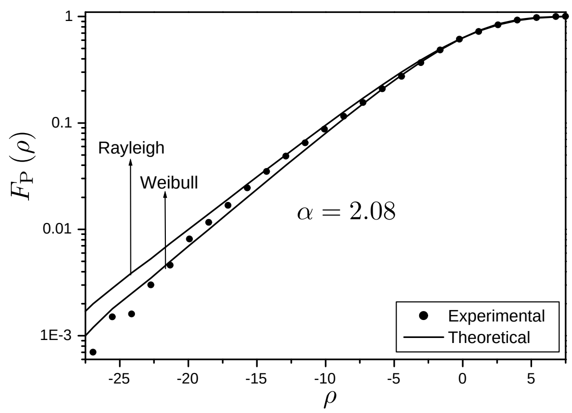

Fig. 4. Empirical versus theoretical CDF

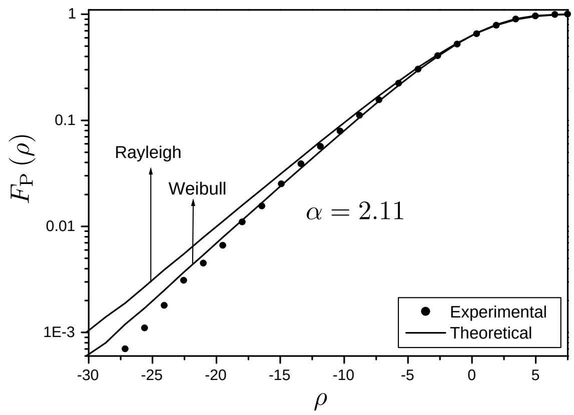

Fig. 5. Empirical versus theoretical CDF

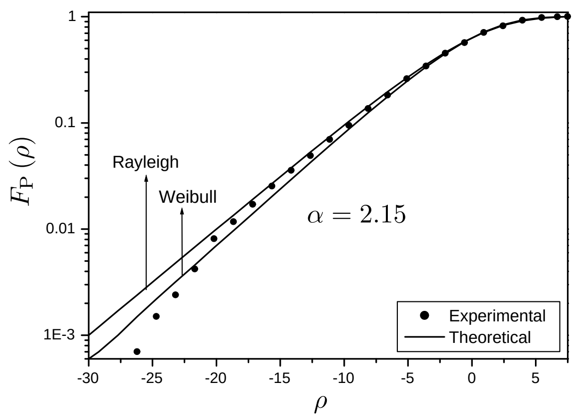

Fig. 6. Empirical versus theoretical CDF